Proposed experiment with Rydberg atoms to test the wave function interpretation

Michel Gondran

*EDF, Research and Development, 92140 Clamart, France**

Mirjana Božic and Dušan Arsenović

Institut of Physics, P.O. Box 57, 11001 Belgrade, Serbia and Montenegro, Yugoslavia†

Alexandre Gondran

SET, Université Technologique de Belfort Montbéliard, France‡

Abstract

Experiment[1] shows that Rydberg atoms do not pass through 1μ m width slits if their principal quantum number is rather large($n \geq 60$). Thus, the particle density measured after the slits is null while the wave function calculated after the slits is not. This experiment is in contradiction with the Born interpretation (the square of the wave function is proportional to the probability density for the particle to be found at each point in space). The classical interpretation of this experiment, which removes the contradiction, is to suppose that if the particles do not pass, the wave function does not pass either (*classical assumption*).

An alternative interpretation of this experiment is to suppose that the wave function passes through the slits, but that the Born interpretation is not valid any more in this case (*alternative assumption*).

The aim of this paper is to present an experiment testing this *alternative assumption* compared to the *classical assumption*.

I. INTRODUCTION

One of the fundamental postulates of quantum mechanics is the Born interpretation, where the square of the module of the wave function is equal to the particle density. However, for a long time, there were no experiments that could separate the particle from its wave function; the wave function accounts for the mass, the wavelength, the spin and the particle electric charge. On the other hand, the wave function does not account for the particle size. However, a quite old experiment of Haroche's team[1] accounts explicitly for Rydberg atoms size. It is an experiment where one measures "*the transmission of a beam of Rydberg atoms through a metallic grating made of an array of micrometre size slits. The transmission decreases linearly with the square of the principal quantum number n, with a cut-off for a maximum n value*". This experiment inspired recently Dahl *et al.*[2] to study theoretically the scattering of a rotor from a slit in the limit when the length of the rotor is larger than the size of the slit.

Thus, as soon as n is rather large ($n \geq 60$ in this experiment for $1\mu m$ width slits), the particle density measured after the slits is null while the wave function calculated after the slits is not. This is in contradiction to the Born interpretation.

The usual interpretation of this experiment, which removes the contradiction, is to suppose that if the particles do not pass, the wave function does not pass either (*classical assumption*). This interpretation, which seems obvious for the majority of the quantum mechanics community, was however never tested in experiments; the presence of the wave function being measured only via the impact of particles.

Although not very probable, one cannot thus eliminate logically alternative interpretation of this experiment which supposes that the wave function passes through the slits but not the particles (*alternative assumption*). In this case, the Born interpretation is not valid.

The aim of this paper is to present an experiment testing this *alternative assumption* compared to the *classical assumption.*

The idea of this experiment is initially based on the fact that diffraction and interference phenomena exist for large particles, cf. Schmiedmayer *et al*[3] and Chapman *et al*[4] for molecules of Na_2 ($\sim 0.6nm$ size) and the Zeilinger team for the fullerene molecules C_{60} ($\sim$ 1 nm diameter)[5], C_{70}[6] and more recently the fluorofullerenes $C_{60}F_{48}$[7]. It is also based on Bozic *et al*[8,10] thought experiments which consider interference with different size slits.

Section 2 describes this test experiment which is an interference experiment with Rydberg atoms and asymmetrical slits; one wide slit A let the Rydberg atoms pass, and one grid B of small slits does not let them pass.

Section 3 gives the value of the square wave function after the slits in the case of both assumptions, the *classical assumption* and the *alternative assumption*.

Finally, in section 4, we present the atomic density which we should measure on the detection screen in the case of the alternative hypothesis. One also supposes in this case that the Rydberg atoms pass through the slit A but do not pass through the grid B.

II. THE RYDBERG ATOMS ASYMMETRIC EXPERIMENT

An asymmetric Rydberg atoms experiment, inspired by the Fabre *et al* experiment[1], is proposed. A Rydberg atoms' beam with a principal quantum number $n = 60$, whose speed is $v_y = 200m/s$ along the 0y axis, is considered. Initial speeds in the other directions are considered null.

The beam is regarded as Gaussian with 6mm width.

At the distance $d_1 = 1$m from the molecular beam source, a metal foil is placed that has a slit A with a 100μm slit width along the axis 0x and a grating B with 1000 small slits of 0.1 μm slit width and 0.2μm slit separation along the 0x axis. The distance between the centers of slit A and grating B is 300μm. Rydberg atoms are then observed by using a detector placed at $d_2 = 2$m behind the slits. The Rydberg atom beam is obtained from sodium atoms, so the atom mass is $m = 3.84\ 10^{-26}$kg. With the velocity $v_y = 200m/s$, the de Broglie wavelength $\lambda_{dB} = 8.6 \times 10^{-5}\mu$m is much smaller than the slits size.

The slit size of the grid B (0.1 μm) is ten times smaller than for the Fabre *et al* experiment[1]. In this case, the Rydberg atoms with n=60 do not pass through the grid. On the other hand, slit A is 100 times larger than the Rydberg atoms' size and thus lets them pass easily.

For n=60, the 70 ms of the Rydberg atom lifetime are sufficient for the experiment (15 ms).

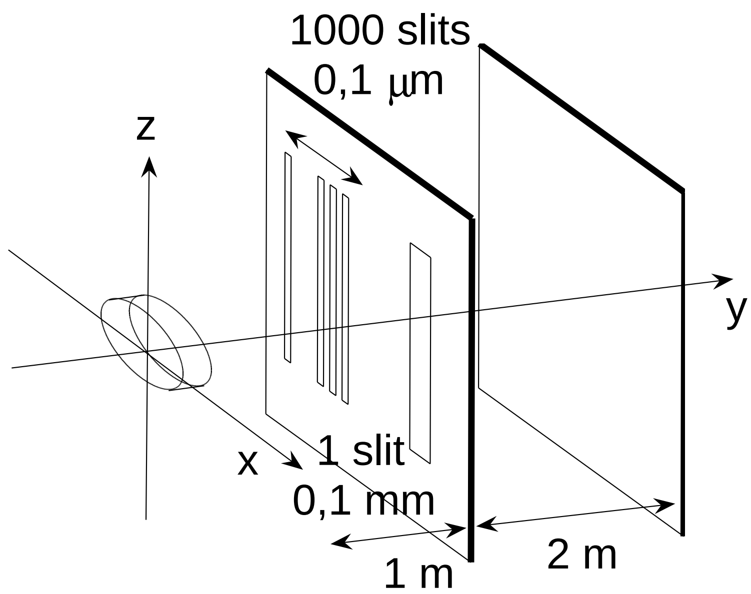

FIG. 1: Schematic diagram of the Rydberg atoms asymmetric experiment.

The case of the classical assumption where the wave function does not pass through the grid B corresponds to diffraction by the slit A.

The case of the alternative assumption where the wave function passes through the grid B corresponds to an interference problem between the slit A and the grid B slits.

III. WAVE FUNCTION CALCULATION WITH FEYNMAN PATH INTEGRAL

The wave function calculation is obtained by a numerical calculation using Feynman integrals, as we did[12] for the numerical simulation of Shimizu *et al* 's experiment with cold atoms.

The wave function before the slits is then equal to

$$\psi(x,t) = (2\pi s(t)^2)^{-\frac{1}{4}} \exp^{-\frac{x^2}{4\sigma_0 s(t)}} \tag{1}$$

with $s(t) = \sigma_0(1 + \frac{i\hbar t}{2m\sigma_0^2})$. After the slits, at time $t \geq t_1 = \frac{d_1}{v_y} = 5\ ms$, the time-dependent wave function $\psi(x,t) = \psi_A(x,t) + \psi_B(x,t)$ is calculated by the Feynman path integral method[11]:

$$\psi_{A/B}(x,t) = \int_{A/B} K(x,t;x_f,t_1)\psi(x_f,t_1)dx_f \tag{2}$$

where $\psi(x_f, t_1)$ is given by (1) and

$$K(x,t;x_f,t_1) = \left(\frac{m}{2i\pi\hbar(t-t_1)}\right)^{\frac{1}{2}} \exp\frac{im}{\hbar}\frac{(x-x_f)^2}{2(t-t_1)}. \tag{3}$$

The integrations in (2) are carried out respectively on the slit A and on the grid B slits.

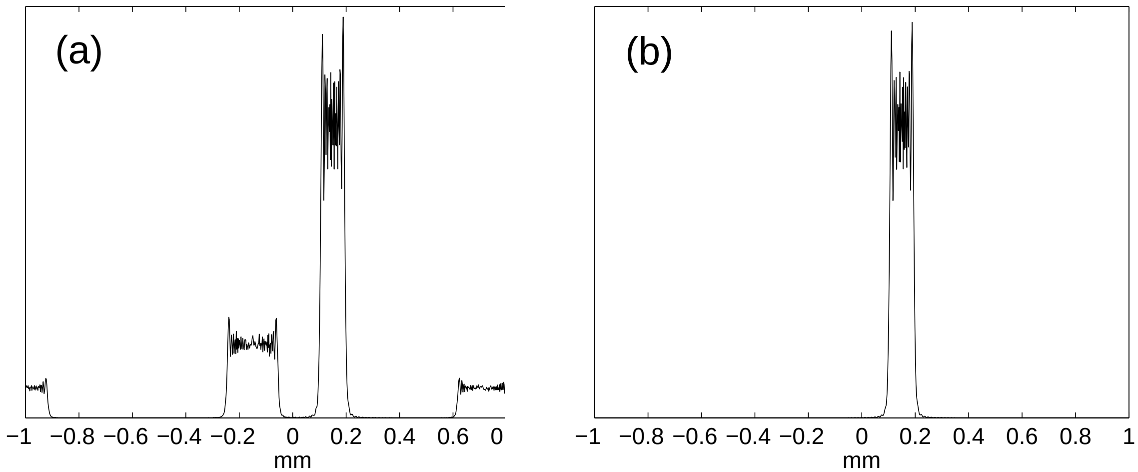

FIG. 2: Square of the wave function $|\psi(x)|^2$ at 2 m from the slits (a) in the case of interference between A and B, and (b) in the case of diffraction by A.

Figure 2 represents the square of the wave function $|\psi(x,t)|^2$ on the detector at 2 m behind the slits for the interference experiment between the slit A and the grating B (figure 2a), and for the diffraction experiment with the slit A only (figure 2b). For the diffraction case, we obtain classically one peak, and for the interference case, we obtain distinctly four peaks.

Figure 3 details the $|\psi(x,t)|^2$ evolution according to the distance to the slits in the case of interference between A and B.

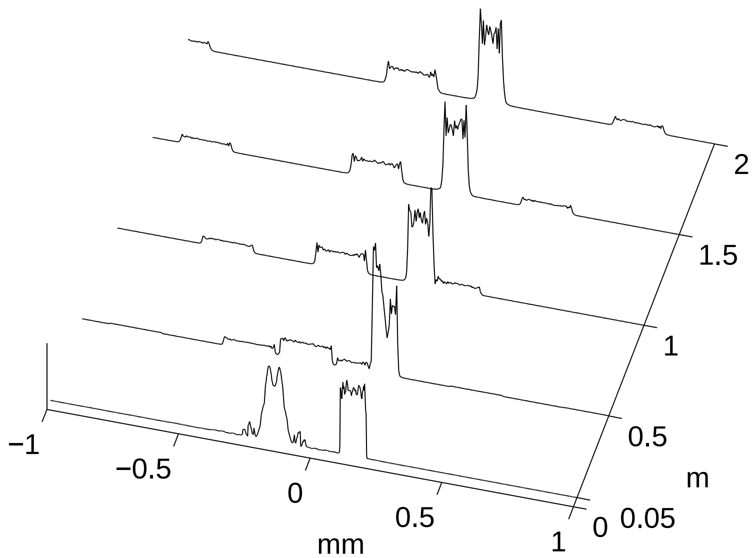

FIG. 3: Evolution of the square wave function $|\psi(x,t)|^2$ in the first 200 cm behind the slits for the case of interference between A and B. Note that $\psi(x,t)$ is equivalent to $\psi(x,y)$ because $t = y/v_y + t_1$ with $v_y = 200m/s$. So calculating the wave function at 2m from the slits is equivalent to calculating the wave function 10ms after it passes through the slits ($y/v_y = 2/200 = 0.01s$)

If the particles size is not considered (*usual assumption*), the wave function and the particles would "pass through" the slit A **and** the grid B. The particle density measured on the detector will then have to correspond to the four peaks of figure 2a (interferences phenomenon between A and B).

In the case of the *classical assumption*, the wave function and the particles "pass through" the slit A **but "do not pass through"** the grid B. The particle density measured on the detector will have to correspond to the single peak of figure 2b (diffraction phenomenon by slit A).

We study in the next paragraph the *alternative assumption* where:

- the wave function "passes through" the slit A **and** the grid B,
- the particles "pass through" the slit A **but "do not pass through"** the slits of grid B.

IV. ALTERNATIVE ASSUMPTION

In the traditional experiments of double slits carried out with molecules whose size is smaller than the slits, the wave function passes through the two slits to form interferences, but each molecule passes through one or the other of the slits, without knowing through which slit it passed. The idea of the alternative assumption is the same. The wave function passes through slit A and grid B and thus will create the interferences of figure 2a.

However, the atoms do not pass through the slits of the grid B. One deduces from this that, just behind the grid B, the probability density of the atoms must be null since no atom crossed it. In the case of the *alternative assumption*, $|\psi(x,t)|^2$ thus no longer represents the probability density of the atoms.

We are in a nonusual situation in quantum mechanics. However, as we already showed in[12], we verify that, just after the slits at time t_1^+, there is not yet covering between the wave function ψ_A which passed by A and the wave function ψ_B which passed by B and we have $|\psi_A(x,t_1^+)+\psi_B(x,t_1^+)|^2=|\psi_A(x,t_1^+)|^2+|\psi_B(x,t_1^+)|^2$.

Since the atoms do not pass by the grid B, one deduces from it that the density $\rho((x,t_1^+)$ of the atoms which passed is equal to $|\psi_A(x,t_1^+)|^2$, the part of the right-hand side of $|\psi(x,t_1^+)|^2$, cf. figure 3 at time $t_1+0.25$ ms (0.05 m).

Since the sum of the widths of the slits of the grid B is equal to the width of slit A, the density $\rho((x,t_1^+)$ is thus

$$\rho(x,t_1^+)=\begin{cases}0 & \text{if} \quad -\infty<x<x_{t_1^+},\\ |\psi(x,t_1^+)|^2 & \text{if} \quad x_{t_1^+}<x<+\infty\end{cases} \tag{4}$$

where $x_{t_1^+}$ is the median of the function $F(x,t)=\int_{-\infty}^{x}\frac{|\psi(x,t)|^2}{\int_{-\infty}^{+\infty}|\psi(u,t)|^2du}du$, i.e. such as $F(x_{t_1^+},t_1^+)=\frac{1}{2}$. By continuity, we can suppose that the atoms, which passed through the slit A, satisfy at each time this property: the density is thus

$$\rho(x,t)=\begin{cases}0 & \text{if} \quad -\infty<x<x_t,\\ |\psi(x,t)|^2 & \text{if} \quad x_t<x<+\infty\end{cases} \tag{5}$$

where x_t is the median of the function $F(x,t)$.

Figure 4 details the density evolution of the atoms having passed throug slit A according to the distance to the slits.

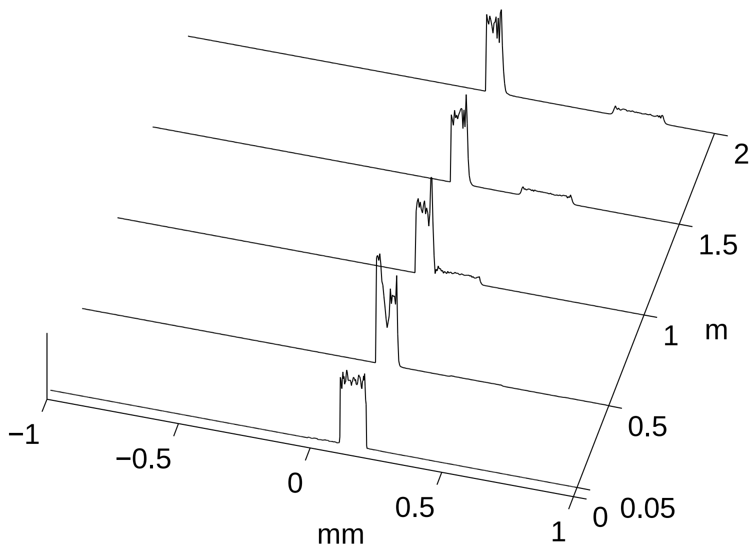

FIG. 4: Evolution on the atoms density in the first 200 cm after slits for the case of the *alternative assumption*.

Consequently, at the time $t_2 = 15$ms, i.e. on the detector, the particle density will be equal to the density located on the right of the median. Figure 5 shows this density corresponding to the right-hand side of figure 2a.

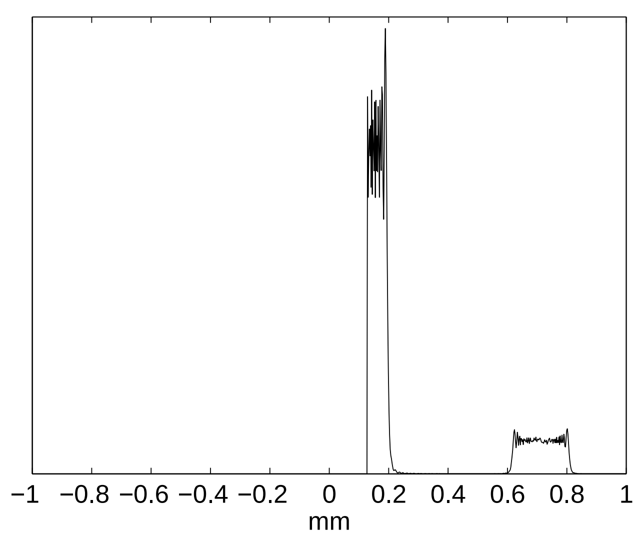

FIG. 5: Particle density at 2 m from the slits in the case of the *alternative assumption*.

Another way of taking into account the alternative assumption assumes the existence of the trajectories. The trajectories may be de Broglie-Bohm trajectories[12,13,14,15,16] or the

trajectories approximately determined by transverse momenta of particles and their distribution[9,10]. In this case, the particles which follow the trajectories and have to pass through the grid B are stopped. The particles having to pass through slit A pass; and their trajectories correspond to the evolution of the densities of figure 4. One thus obtains the same result as figure 5, but with an assumption unnecessary.

The clear difference between the density of figures 2b and 5 shows that the asymetric Rydberg atoms experiment can be a very good test between the *classical assumption* and the *alternative assumption*. This test is robust in spite of the variation of the initial data with the initial speeds v_x, v_y and v_z. We have shown[12] how to take these uncertainties into account. They will smooth the densities, but will preserve the number of peaks.

V. CONCLUSION

Taking into account the size of the Rydberg atoms in the interference phenomena makes it possible to renew the study of the wave-particle duality and to propose an experiment to test the wave function interpretation.

One tests the traditional interpretation that if the particles do not pass through a slit, the wave function does not pass either, against the *alternative assumption* that the wave function passes through the slit but that the interpretation of Born is not valid any more in this case.

This experiment appears realizable today, and each of the two alternatives will contribute an interesting contribution to the interpretation of the wave function. Indeed, even in the case considered as most probable where the experiment confirms the *classical assumption*, this result will have to be to add to the other postulates of quantum mechanics.

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* Electronic address: michel.gondran@chello.fr

† Electronic address: bozic@phys.bg.ac.yu, arsenovic@phys.bg.ac.yu

‡ Electronic address: alexandre.gondran@utbm.fr

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